A SEMI-ANALYTICAL & SEMI-NUMERICAL METHOD FOR SEISMIC ANALYSIS OF ARCH DAM

by

Cao Zhi-yuan, Associate Research Prof.
Institute of Engineering Mechanics, Academia Sinica, China
Zhang Yao-qin, Research Associate,
Zhejiang Institute of Computing Technique, China

ABSTRACT

A semi-analytical finite element method, which makes use of simple polynomials only in transverse direction and analytical functions in the vertical and thickness directions, for seismic analysis of arch dam is presented. The solution of using thick shell analytical functions is equivalent to the numerical calculation of a discrete system in vertical and thickness directions, hence the computational effort and the core requirements are reduced considerably. In this paper a practical arch dam in China with 69 m in height, under San Fernando (1971) earthquake was calculated by two methods: the semi-analytical method and finite element method, in addition the test results of natural frequences of this dam are given for comparision. The advantages of the present approach in higher accuracy and fewer degrees of freedom than finite element method are shown.

INTRODUCTION

The dynamic analysis of dam is an important problem in earthquake engineering. It is a three-dimentional dynamic problem and used to be solved by numerical method. As well known, the finite element method (F.E.M.) is a powerful and versatile tool of solution in structural analysis including seismic analysis of structures. However, the cost of solutions can be very expensive, when a multi-dimensional analysis is required. Therefore the recently developed finite strip method (F.S.M.) (1), which can reduce the computational effort and the core requirements, is desirable for such cases. But for some three dimensional structures (such as arch dam), the computational effort is still large, when the ordinary finite strip method is applied. Here we proposed a semi-analytical finite element method (S.F.E.M.), which is suitable for some three dimensional structures, such as arch dam.

In the sense of mechanics the arch dam is an irreqular thick shell with variable thickness. In this paper, the semi-analytical finite element method makes use of simple polynomials in transverse direction and analytical functions in vertical and thickness directions. In the practical solution we choice the analytical solution of mode shapes of flat thick shell to construct the displacement functions and the arch dam is divided into some vertical cantilever shell strips. The solution of using thick shell analytical functions is equivalent to the numerical calculation of a discrete system in vertical and thickness directions, hence the computational effort and the core requirements are reduced considerably. In addition, because this type of analytical function is in itself an exact solution of one-dimensional problem of the corresponding structure, the present method has higher accuracy and better convergence (2,3) than the ordinary finite strip method.

THE MODE SHAPE FUNCTIONS OF FLAT THICK SHELL STRIP

For the dynamic problems of flat thick shell strip we have the fundamental equations including shear and rotatory inertia (4)

$$\frac{\partial^4 W}{\partial y^4} - \left(\frac{\rho h}{Gh} + \frac{\rho J}{D}\right)\frac{\partial^4 W}{\partial y^2 \partial t^2} + \frac{\rho h}{Gh} \times \frac{\rho J}{D}\frac{\partial^4 W}{\partial t^4} + \frac{\rho h}{D}\frac{\partial^2 W}{\partial t^2} = \frac{q}{D} + \frac{1}{Gh}\left(\frac{\rho J}{D}\frac{\partial^2 q}{\partial t^2} - \frac{\partial^2 q}{\partial y^2}\right) \qquad (1.1)$$

$$Gh\beta_y + \rho J\frac{\partial^2 \beta_y}{\partial t^2} + D\frac{\partial^3 W}{\partial y^3} + Gh\frac{\partial W}{\partial y} - \frac{\rho h}{Gh}D\frac{\partial^3 W}{\partial y \partial t^2} = -\frac{D}{Gh}\frac{\partial q}{\partial y} \qquad (1.2)$$

and relation

$$\rho h\frac{\partial^2 w}{\partial t^2}-Gh\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial \beta_y}{\partial y}\right)=q \tag{2.1}$$

where $w(y,t)$ is the deflection, $\beta_y(y,t)$ is the rotatory angle (the origin of the coordinate y is assumed to be on the side of the strip), ρh, ρJ are the relative mass and relative rotatory inertia of strip; D,Gh are the strip rigidity in bending and in shear, respectively, $q(y,t)$ is the relative loading.

For the bending moment and shear force the following formulae apply

$$M_y=D\frac{\partial \beta_y}{\partial y} \tag{2.2}$$

$$Q_y=Gh\left(\beta_y+\frac{\partial w}{\partial y}\right) \tag{2.3}$$

The normal modes of vibration $Y(y)$, $\Phi(y)$ are determined from the mode shape equations corresponding to eqn. (1)

$$\frac{d^4Y}{dy^4}+\left(\frac{\rho h}{Gh}+\frac{\rho J}{D}\right)\omega^2\frac{d^2Y}{dy^2}+\left(\frac{\rho h}{Gh}\cdot\frac{\rho J}{D}\omega^4-\frac{\rho h}{D}\omega^2\right)Y=0 \tag{3.1}$$

$$\left(\frac{Gh}{D}-\frac{\rho J}{D}\omega^2\right)\Phi+\left(\frac{\rho h}{Gh}\omega^2+\frac{Gh}{D}\right)\frac{dY}{dy}+\frac{d^3Y}{dy^3}=0 \tag{3.2}$$

where ω is the natural frequency of strip.

The solution of homogeneous eqn. (3) may be written in the form

$$Y(y)=A_0\sin\alpha_0 y+B_0\cos\alpha_0 y+C_0\,\mathrm{sh}\,\alpha_1 y+D_0\,\mathrm{ch}\,\alpha_1 y \tag{4.1}$$

$$\begin{aligned}\Phi(y)=\;&A_0(\beta_1\alpha_0-\beta_0\alpha_0^3)\cos\alpha_0 y+B_0(\beta_0\alpha_0^3-\beta_1\alpha_0)\sin\alpha_0 y\\&+C_0(\beta_1\alpha_1+\beta_0\alpha_1^3)\,\mathrm{ch}\,\alpha_1 y+D_0(\beta_1\alpha_1+\beta_0\alpha_1^3)\,\mathrm{sh}\,\alpha_1 y\end{aligned} \tag{4.2}$$

where

$$\begin{matrix}\alpha_0\\\alpha_1\end{matrix}=\frac{\bar{\omega}}{\sqrt{2}\,l}\sqrt{\sqrt{(I_G-I_\rho)^2+\left(\frac{2}{\bar{\omega}}\right)^2}\pm(I_G+I_\rho)} \tag{5.1}$$

$$\beta_0=l^2/(I_\rho\bar{\omega}^2-1/I_G) \tag{5.2}$$

$$\beta_1=(I_G\bar{\omega}^2+1/I_G)/(I_\rho\bar{\omega}^2-1/I_G) \tag{5.3}$$

$$\bar{\omega}=\omega l^2\sqrt{\rho h/D} \tag{5.4}$$

$$I_G=D/Ghl^2 \tag{5.5}$$

$$I_\rho=\rho J/\rho h l^2 \tag{5.6}$$

and l is the strip length in direction of y-axis.

The integration constants A_0, B_0, C_0, D_0 are determined from the boundary conditions on two sides. For the cantilever strip in the vertical direction of the arch dam, considering eqn. (2), the boundary conditions has the form

$$y=0\;,\quad w=0\;,\quad \text{i.e.}\quad Y=0 \tag{6.1}$$

$$\beta_y=0\ ,\quad \text{i.e.}\quad \Phi=0 \tag{6.2}$$

$$y=1\ ,\quad M=0\ ,\quad \text{i.e.}\quad Y''+\frac{I_q}{I_p}\bar{\omega}^2 Y=0 \tag{6.3}$$

$$Q=0\ ,\quad \text{i.e.}\quad \Phi+Y'=0 \tag{6.4}$$

The above conditions for y=0,y=1 give four homogeneous equtions for the calculation of integration constants. The condition for the determinant of this system to be zero gives a frequency equation, which is transcendental and has infinite numbers of roots $\bar{\omega}_m$ and corresponding α_{0m}, α_{1m}. Substituting these solutions into the homogeneous eqution system for integration constants and solving them, we obtain the shape of m-th mode for the cantilever thick strip.

$$Y_m(y)=\sin\alpha_{0m}y-b_m\cos\alpha_{0m}y+a_m\,\mathrm{sh}\,\alpha_{1m}y+b_m\,\mathrm{ch}\,\alpha_{1m}y \tag{7.1}$$

$$\Phi_m(y)=c_m b_m\sin\alpha_{0m}y+c_m\cos\alpha_{0m}y+d_m b_m\,\mathrm{sh}\,\alpha_{1m}y+d_m a_m\,\mathrm{ch}\,\alpha_{1m}y \tag{7.2}$$

where

$$a_m=(\alpha_{0m}l)[(\alpha_{0m}l)^2-I_q\bar{\omega}_m^2-1/I_q]/(\alpha_{1m}l)[(\alpha_{1m}l)^2+I_q\bar{\omega}_m^2+1/I_q] \tag{8.1}$$

$$b_m=\frac{[I_q\bar{\omega}_m^2-(\alpha_{0m}l)^2]\sin\alpha_{0m}l+a_m[I_q\bar{\omega}_m^2+(\alpha_{1m}l)^2]\,\mathrm{sh}\,\alpha_{1m}l}{[I_q\bar{\omega}_m^2-(\alpha_{0m}l)^2]\cos\alpha_{0m}l-[I_q\bar{\omega}_m^2+(\alpha_{1m}l)^2]\,\mathrm{ch}\,\alpha_{1m}l} \tag{8.2}$$

$$c_m=[(I_q\bar{\omega}_m^2+1/I_q)(\alpha_{0m}l)-(\alpha_{0m}l)^3]/(I_p\bar{\omega}_m^2-1/I_q) \tag{8.3}$$

$$d_m=[(I_q\bar{\omega}_m^2+1/I_q)(\alpha_{1m}l)+(\alpha_{1m}l)^3]/(I_p\bar{\omega}_m^2-1/I_q) \tag{8.4}$$

THE SEMI-ANALYTICAL FINITE ELEMENT METHOD

Before all, the arch dam may be divided into cantilever shell strips with variable thickness along vertical direction. In accordance with theoretical analysis (5,6), each strip (Fig.1) has five displacement components, which can be written as

$$u(\xi,y)=\sum_{m=1}^{r}[(1-\xi)u_{1m}+\xi u_{2m}]Y_m(y) \tag{9.1}$$

$$v(\xi,y)=\sum_{m=1}^{r}[(1-\xi)v_{1m}+\xi v_{2m}]\Phi_m(y) \tag{9.2}$$

$$w(\xi,y)=\sum_{m=1}^{r}[(1-\xi)w_{1m}+\xi w_{2m}]Y_m(y) \tag{9.3}$$

$$\beta_x(\xi,y)=\sum_{m=1}^{r}[(1-\xi)\theta_{x1m}+\xi\theta_{x2m}]Y_m(y) \tag{9.4}$$

$$\beta_y(\xi,y)=\sum_{m=1}^{r}[(1-\xi)\theta_{y1m}+\xi\theta_{y2m}]\Phi_m(y) \tag{9.5}$$

or in matrix form

$$\{f\}=\sum_{m=1}^{r}[N]_m\{\delta\}_m=[N]\{\delta\} \tag{10}$$

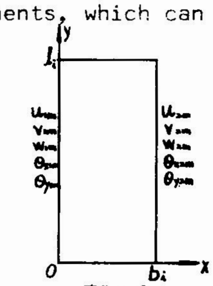

Fig.1

where $Y_m(y)$, $\Phi_m(y)$ are the m-th order of analytical solutions of mode shape equation of thick strip as above, u_{1m}, v_{1m}

$,w_{1m},\ldots\ldots\theta_{y,m}$ are the displacement and rotation parameters at the two edges for the m-th term of the series and

$$\xi = x/b_i \qquad (11)$$

According to thick shell theory, there are eight strain components in the shell strip (7)

$$\{\varepsilon\} = [\,\varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy} \;\; \chi_x \;\; \chi_y \;\; \chi_{xy} \;\; \gamma_{xz} \;\; \gamma_{yz}\,]^T$$
$$= [\tfrac{\partial u}{\partial x} \;\; \tfrac{\partial v}{\partial y} \;\; \tfrac{\partial u}{\partial y}+\tfrac{\partial v}{\partial x} \;\; \tfrac{\partial \beta_x}{\partial x} \;\; \tfrac{\partial \beta_y}{\partial y} \;\; \tfrac{\partial \beta_x}{\partial y}+\tfrac{\partial \beta_y}{\partial x} \;\; \beta_x+\tfrac{\partial w}{\partial x} \;\; \beta_y+\tfrac{\partial w}{\partial y}]^T \qquad (12.1)$$

Substituting eqn. (9) into strain expressions, we obtain the strain matrix

$$\{\varepsilon\} = \sum_{m=1}^{r} [B]_m \{\delta\}_m = [B]\{\delta\} \qquad (12.2)$$

For the axial forces, bending and torsional moments and shear forces we have the internal force matrix (7)

$$\{Q\} = [\,N_x \; N_y \; N_{xy} \; M_x \; M_y \; M_{xy} \; Q_x \; Q_y\,]^T = [D(y)]\{\varepsilon\} \qquad (13.1)$$

where the rigidity matrix

$$[D(y)] = \begin{bmatrix} [D_1(y)] & [0] \\ [0] & [D_2(y)] \end{bmatrix} \qquad (13.2)$$

$$[D_1(y)] = \frac{Eh(y)}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \qquad (13.3)$$

$$[D_2(y)] = \frac{Eh^3(y)}{12(1-\mu^2)}\begin{bmatrix} 1 & \mu & 0 & 0 & 0 \\ \mu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\mu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{6(1-\mu)}{h^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{6(1-\mu)}{h^2} \end{bmatrix} \qquad (13.4)$$

E is the modulus of elasticity, μ is the Poïssion coefficient, h(y) is the strip thickness.

Using variational principle, the stiffness and mass matrices in local coordinate system may be written as

$$[K'] = \int [B]^T [D(y)] [B] dV \qquad (14.1)$$
$$[M'] = \int [N]^T [\rho(y)] [N] dV \qquad (14.2)$$

where

$$[\rho(y)] = \begin{bmatrix} \rho h(y) & & & & 0 \\ & \rho h(y) & & & \\ & & \rho h(y) & & \\ & & & \rho J(y) & \\ 0 & & & & \rho J(y) \end{bmatrix} \qquad (14.3)$$

By transformation the stiffness, mass, load matrices in common coordinate system can be obtained as

$$[K] = [R][K'][R]^T \qquad (15.1)$$

$$[M]=[R][M'][R]^T \qquad (15.2)$$

$$\{F(t)\}=\int[N]^T\{q(t)\}dV \qquad (15.3)$$

in which [R] is the transformation matrix, $\{q(t)\}$ consists of the load and moment terms. For example, for three-dimentional seismic loading, the ground motion has three components u_g, v_g, w_g and the m-th term of load matrix

$$\{F\}_m=-\frac{\rho b_i}{2}[I_1\ddot{u}_g \; I_2\ddot{v}_g \; I_1\ddot{w}_g \; 0 \; 0 \; I_1\ddot{u}_g \; I_2\ddot{v}_g \; I_1\ddot{w}_g \; 0 \; 0]^T \qquad (16.1)$$

$$I_1=\int h(y)Y_m(y)dy \; , \quad I_2=\int h(y)\Phi_m(y)dy \qquad (16.2)$$

In the end, introducing damping matrix, the seismic analysis of arch dam becomes the solution to one-dimentional discretization equation

$$[M]\{\ddot{\delta}(t)\}+[C]\{\dot{\delta}(t)\}+[K]\{\delta(t)\}=\{F(t)\} \qquad (17)$$

And the displacement, strain, stress components of the arch dam may be obtained by using eqn. (9),(12),(13).

NATURAL FREQUENCY AND SEISMIC ANALYSIS OF ARCH DAM

The Heng-Shen arch dam in China with 69 m in height, 146 m in length along dam crest, 15 m in thickness on the dam ground is divided into 12 strips and has 13 nodal lines (Fig.2).

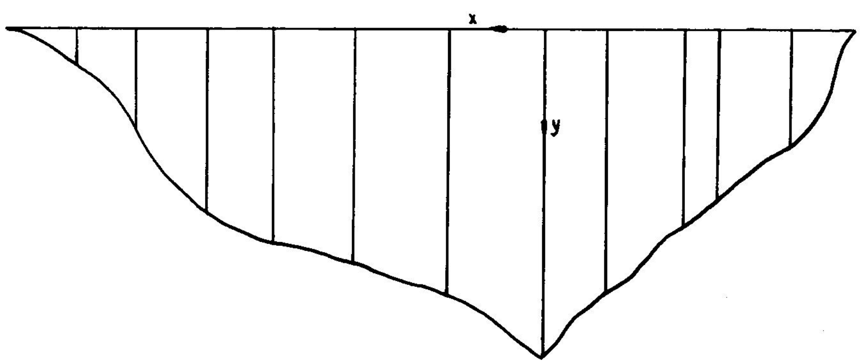

Fig.2

The natural frequencies and seismic response of the dam with Rayleigh damping matrix $\alpha=1.152$, $\beta=0.00078$ to San Fernando (1971) earthquake with 0.1 g peak horizontal acceleration were calculated by the present method and by 3-D finite element method. The numerical results of the first eight frequences with the comparison with test results (8); the max. seismic deflection and stresses of the dam; and the DOF and cost are given in tables 1,2,3, respectively. From these tables it is seen that the semi-analytical finite element has the same accuracy as the conventional finite element method, but has fewer elements, DOF, core and computational effort than the later.

Table 1. Natural Frequencies of the Arch Dam

Order	S.F.E.M. (Rigid ground)	F.E.M. (Rigid ground)	Test (Elastic ground)	
			Model	In-Situ.
ω_1	41.12	41.25	42.10	37.07
ω_2	44.75	44.59	44.61	39.58
ω_3	57.71	59.87	55.92	-
ω_4	76.91	71.56	62.83	-
ω_5	78.67	80.41	65.97	-
ω_6	102.6	101.0	98.65	74.77
ω_7	106.1	111.8	-	-
ω_8	116.6	115.5	-	-

Table 2. Seismic Response of the Arch Dam

Method	Max. deflection(cm)	Max. stress (kg/cm^2)		
	w	σ_x	σ_y	τ_{xy}
S.F.E.M.	0.258	3.20	2.05	1.89
F.E.M.	0.244	3.66	1.95	1.98

Table 3. Comparison of Cost between S.F.E.M. and F.E.M.

Method	Elements	DOF	Core	Comp. effort
S.F.E.M.	12	55	9000	62minx1.2x10^{5}/sec
F.E.M.	102	534	85000	118minx10x10^{3}/sec

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